Solution to Exercise 9.9 (Version 1, 1/8/15)

from Statistical Methods in Biology: Design & Analysis of Experiments and Regression (2014) S.J. Welham, S.A. Gezan, S.J. Clark & A. Mead. Chapman & Hall/CRC Press, Boca Raton, Florida. ISBN: 978-1-4398-0878-8

Exercise 9.9

An experiment to compare yields of 13 varieties of corn was set up as a BIBD with 13 blocks, each containing four plots (Cochran & Cox, 1957, Table 11.2). File CORN.DAT contains the unit numbers (*ID*), structural factors (Block, Plot), explanatory factor (Variety) and plot yields (variate *Yield*, pounds per plot). Calculate λ and the efficiency factor for this design. Is the design resolvable? Is there any evidence of differences in yield among the varieties?

Data 9.9 (CORN.DAT)

Yields (pounds per plot) for 13 corn varieties from a BIBD with 13 blocks of four plots:

ID	Block	Plot	Variety	Yield	ID	Block	Plot	Variety	Yield
1	1	1	10	16.2	27	7	3	13	35.3
2	1	2	13	26.6	28	7	4	9	26.9
3	1	3	11	19.3	29	8	1	2	30.3
4	1	4	12	31.7	30	8	2	6	31.5
5	2	1	11	24.6	31	8	3	7	39.3
6	2	2	6	19.9	32	8	4	12	26.7
7	2	3	9	29.0	33	9	1	2	32.9
8	2	4	3	25.3	34	9	2	3	37.3
9	3	1	6	33.7	35	9	3	1	38.2
10	3	2	1	31.8	36	9	4	10	31.3
11	3	3	13	41.1	37	10	1	9	25.7
12	3	4	8	27.8	38	10	2	1	34.7
13	4	1	8	35.6	39	10	3	12	30.5
14	4	2	11	17.4	40	10	4	5	31.1
15	4	3	5	27.0	41	11	1	3	23.0
16	4	4	2	27.3	42	11	2	12	22.7
17	5	1	10	32.4	43	11	3	8	33.3
18	5	2	8	30.5	44	11	4	4	19.8
19	5	3	9	30.8	45	12	1	1	36.6
20	5	4	7	23.4	46	12	2	11	28.4
21	6	1	10	32.8	47	12	3	4	31.1
22	6	2	6	27.2	48	12	4	7	31.1
23	6	3	5	32.4	49	13	1	5	32.4
24	6	4	4	30.6	50	13	2	3	34.4
25	7	1	4	30.7	51	13	3	7	33.3
26	7	2	2	28.7	52	13	4	13	36.9

Source: Cochran, W.G. & Cox, G.M. (1957) Experimental designs (2nd Edition). J. Wiley & Sons, New York.

Solution 9.9

For this BIBD experiment, we have $t = 13$ treatments (varieties), $m = 13$ blocks, and $u = 4$ plots per block. There are four replicates of each variety, n, which also can be calculated as

$$n = \frac{m \times u}{t} = \frac{13 \times 4}{13} = 4.$$

The number of times each pair of treatments occurs together within blocks, parameter λ, is calculated as

$$\lambda = \frac{n(u-1)}{(t-1)} = \frac{4 \times 3}{12} = 1.$$

Hence, each pair of varieties occurs together in just one block. It is straightforward to verify this in the data table above, for example, varieties 1 and 2 occur together only in block 9. The efficiency factor for the design is calculated as

$$\text{EF} = \frac{\lambda \times t}{n \times u} = \frac{1 \times 13}{4 \times 4} = \frac{13}{16} = 0.8125.$$

Hence, 81.25% of the information on treatment differences is available from within-block comparisons (the intra-block anaysis). The remainder (18.75%) is available from comparisons across blocks, i.e. from the inter-block analysis. As $t / u = 13 / 4$ is not an integer value, the design is not resolvable. Again, this is easily verified from the data table. The model for the corn yields can be written in symbolic form as

Response variable:	*Yield*
Explanatory component:	[1] + Variety
Structural component:	Block / Plot

Residual plots from the intra-block analysis for this model are in Figure S9.9.1 and show no evidence of variance heterogeneity or a non-Normal distribution.

Table S9.9.1 Multi-stratum ANOVA table for corn yields.

Source of variation	df	Sum of squares	Mean square	Variance ratio	*P*
Block stratum					
Variety	12	689.38	57.45	–	–
Block.Plot stratum					
Variety	12	328.55	27.38	1.37	0.238
Residual	27	538.22	19.93		
Total	51	1556.15			

The ANOVA table is in Table S9.9.1. As we have $t = m$ here, the residual df for the Block stratum is zero, and the variance ratio for treatments cannot be formed within that stratum. As there is relatively little information in the inter-block stratum, this does not compromise the analysis greatly. The variance ratio and observed significance level for the variety term in the Block.Plot stratum ($P = 0.238$) give no evidence of any difference in population means among these varieties.

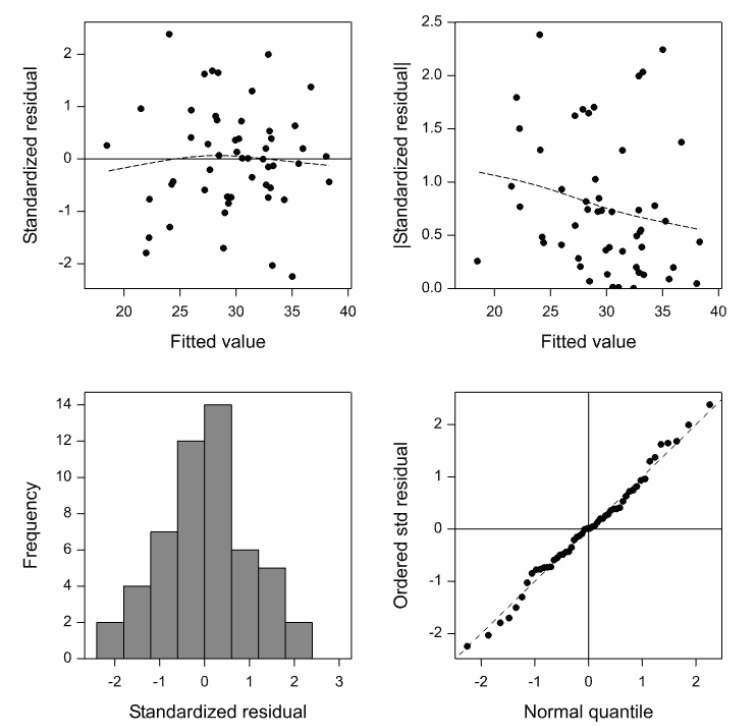

Figure S9.9.1. Composite set of residual plots based on standardized residuals from intra-block analysis.

Table S9.9.2 Mean corn yields (pounds per plot) estimated from the intra-block analysis and combined from both strata.

Variety	Intra-block analysis	Combined estimate
1	33.00	34.17
2	28.27	29.04
3	30.22	30.11
4	28.10	28.08
5	29.96	30.34
6	27.10	27.59
7	29.72	30.76
8	33.72	32.75
9	29.02	28.56
10	28.02	28.10
11	24.52	23.47
12	30.09	28.99
13	35.38	35.18
SED	3.502 (27 df)	3.333 (33.36 df)

The predicted variety yields estimated from the intra-block analyses are shown with the estimates combined across strata in Table S9.9.2. The additional information provided by the inter-block analysis is reflected in smaller SEDs for the combined estimates, and so in practice we would usually prefer these combined estimates. Note that although the range of variety predicted means seems large compared to the SED (or LSD), if we use a sensible multiple comparison test (such as Tukey's simultaneous confidence intervals) we find no evidence of differences within the set.