Solution to Exercise 9.6 (Version 1, 1/8/15)

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Exercise 9.6 (Data: courtesy J. Jenkyn, Rothamsted Research)

A field experiment studied forms and rates of nutrient application and the effect on the yield of spring barley in the presence or absence of foliar diseases. Nitrogen fertilizer was applied either in a liquid form, alone (L) or with a nitrification inhibitor added (LI), or in a solid form, to the seedbed (SS) as a top-dressing (ST) or split (half to the seedbed and half as top-dressing, SST). Each form was applied at two rates (70 and 110 kg N/ha), giving 10 nutrient treatments in total. The occurrence of foliar diseases was intended to be manipulated by a 2 × 2 factorial in presence or absence of a mildew fungicide (None, Tridemorph) and a rust fungicide, but no rust developed and so the latter fungicide was not applied. The trial used a split-plot design with two blocks. The 10 nutrient treatments were applied to main plots, each of which was split into four subplots, and the mildew fungicide was applied to two subplots in each main plot. The plot numbers (*ID*), structural factors (Block, MainPlot, Subplot), explanatory factors (NForm, NRate, MildewF) and yield at harvest (variate *Yield*, tonnes/hectare at 85% dry matter) are in file SPRINGBARLEY.DAT. Identify a suitable predictive model and comment on the comparison between liquid and solid forms of fertilizer.

Data 9.6 (SPRINGBARLEY.DAT)

Yields for spring barley experiment from a split-plot design with 2 blocks, each split into 10 main plots (MP), which are each split into 4 subplots (SP). Treatments were nitrogen rates (NRate) applied in different forms (NForm) in combination with mildew fungicide (MildewF).

Block	MP	SP	NRate	NForm	MildewF	Yield	Block	MP	SP	NRate	NForm	MildewF	Yield
1	1	1	110	LI	Tridemorph	4.62	2	1	1	110	SS	NONE	5.03
1	1	2	110	LI	NONE	4.92	2	1	2	110	SS	NONE	4.94
1	1	3	110	LI	NONE	4.93	2	1	3	110	SS	Tridemorph	5.55
1	1	4	110	LI	Tridemorph	5.03	2	1	4	110	SS	Tridemorph	5.55
1	2	1	70	L	Tridemorph	5.51	2	2	1	70	SS	Tridemorph	5.73
1	2	2	70	L	Tridemorph	5.83	2	2	2	70	SS	NONE	4.77
1	2	3	70	L	NONE	4.61	2	2	3	70	SS	NONE	4.85
1	2	4	70	L	NONE	4.72	2	2	4	70	SS	Tridemorph	5.24
1	3	1	70	LI	Tridemorph	5.53	2	3	1	110	SST	NONE	4.97
1	3	2	70	LI	Tridemorph	5.27	2	3	2	110	SST	NONE	5.02
1	3	3	70	LI	NONE	4.79	2	3	3	110	SST	Tridemorph	5.83
1	3	4	70	LI	NONE	4.97	2	3	4	110	SST	Tridemorph	5.56
1	4	1	110	SST	NONE	4.84	2	4	1	70	LI	Tridemorph	5.52
1	4	2	110	SST	Tridemorph	6.11	2	4	2	70	LI	Tridemorph	5.49
1	4	3	110	SST	NONE	5.46	2	4	3	70	LI	NONE	5.33
1	4	4	110	SST	Tridemorph	6.11	2	4	4	70	LI	NONE	4.94
1	5	1	110	SS	Tridemorph	5.59	2	5	1	70	SST	Tridemorph	5.72
1	5	2	110	SS	Tridemorph	5.76	2	5	2	70	SST	NONE	5.49
1	5	3	110	SS	NONE	4.93	2	5	3	70	SST	Tridemorph	5.83

Block	MP	SP	NRate	NForm	MildewF	Yield	Block	MP	SP	NRate	NForm	MildewF	Yield
1	5	4	110	SS	NONE	4.94	2	5	4	70	SST	NONE	4.80
1	6	1	70	SS	Tridemorph	5.52	2	6	1	110	ST	NONE	4.88
1	6	2	70	SS	NONE	4.82	2	6	2	110	ST	Tridemorph	5.66
1	6	3	70	SS	NONE	4.41	2	6	3	110	ST	NONE	5.05
1	6	4	70	SS	Tridemorph	4.60	2	6	4	110	ST	Tridemorph	5.60
1	7	1	110	ST	Tridemorph	5.95	2	7	1	110	L	NONE	4.97
1	7	2	110	ST	Tridemorph	5.94	2	7	2	110	L	Tridemorph	5.65
1	7	3	110	ST	NONE	4.36	2	7	3	110	L	NONE	5.26
1	7	4	110	ST	NONE	4.47	2	7	4	110	L	Tridemorph	5.62
1	8	1	110	L	Tridemorph	5.49	2	8	1	70	ST	NONE	5.04
1	8	2	110	L	NONE	4.92	2	8	2	70	ST	Tridemorph	5.45
1	8	3	110	L	NONE	4.87	2	8	3	70	ST	NONE	4.76
1	8	4	110	L	Tridemorph	5.56	2	8	4	70	ST	Tridemorph	6.04
1	9	1	70	ST	Tridemorph	5.03	2	9	1	70	L	Tridemorph	5.46
1	9	2	70	ST	NONE	5.66	2	9	2	70	L	NONE	5.10
1	9	3	70	ST	Tridemorph	5.20	2	9	3	70	L	Tridemorph	5.32
1	9	4	70	ST	NONE	4.73	2	9	4	70	L	NONE	5.00
1	10	1	70	SST	Tridemorph	6.03	2	10	1	110	LI	Tridemorph	6.22
1	10	2	70	SST	NONE	5.13	2	10	2	110	LI	Tridemorph	6.03
1	10	3	70	SST	Tridemorph	5.69	2	10	3	110	LI	NONE	5.13
1	10	4	70	SST	NONE	4.70	2	10	4	110	LI	NONE	5.52

Solution 9.6

The split-plot design is a nested structure of subplots (factor Subplot) within main plots (factor MainPlot) within blocks (factor Block). The treatment structure is a 3-way crossed structure of nitrogen rate (factor NRate) with form of application (factor NForm) and presence or absence of mildew fungicide (factor MildewF). The form and application method of fertilizer combinations are applied to main plots and the fungicide is applied (or not) to subplots. With yield (variate *Yield*) as the response, the model can be written as

Response variable:	*Yield*
Explanatory component:	[1] + NRate * NForm * MildewF
Structural component:	Block / MainPlot / Subplot

This model gives the residual plots in Figure S9.6.1. There is one standardised residual > 3 which appears outside of the distribution on the histogram and fitted values plots. However, this residual is part of the trend in the Normal plot and so overall does not cause undue alarm. In practice, we would go back to the data set to double-check for any errors associated with this (or any other) values. There are no strong systematic patterns in the plots, and so we accept the analysis without transformation. The ANOVA table from this model is shown in Table S9.6.1.

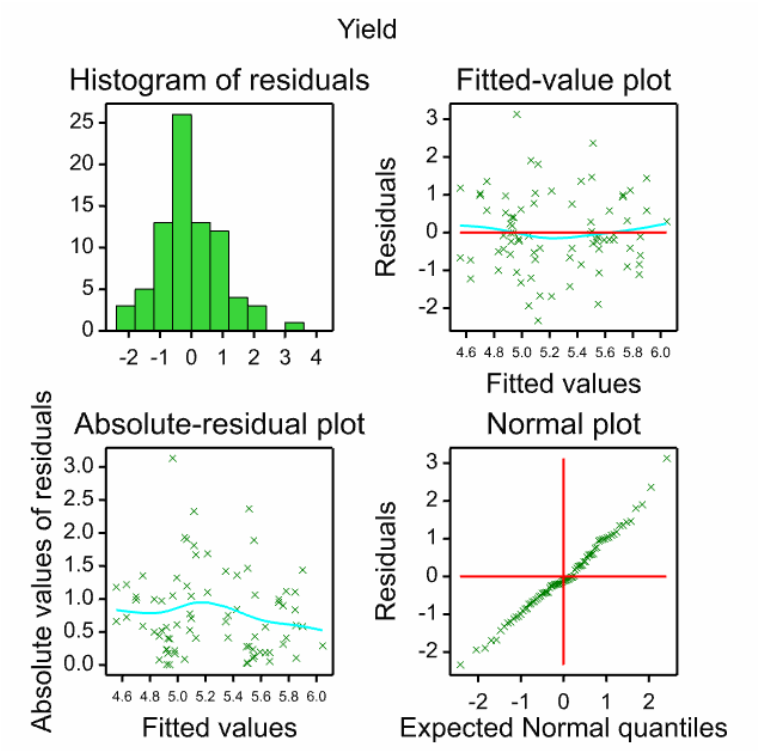

Figure S9.6.1. Composite set of residual plots based on standardized (std) residuals.

Table S9.6.1 ANOVA table for yield of spring barley in split-plot field trial.

Source of variation	df	Sum of squares	Mean square	Variance ratio	*P*
Block stratum	1	0.5072	0.5072		
Block.MainPlot stratum					
NRate	1	0.2216	0.2216	1.317	0.281
NForm	4	0.8517	0.2129	1.266	0.352
NRate.NForm	4	0.1985	0.0496	0.295	0.874
Residual	9	1.5143	0.1683	2.130	
Block.MainPlot.Subplot stratum					
MildewF	1	8.0709	8.0709	102.182	<0.001
NRate.MildewF	1	0.0865	0.0865	1.096	0.300
NForm.MildewF	4	0.3943	0.0986	1.248	0.303
NRate.NForm.MildewF	4	0.4537	0.1134	1.436	0.236
Residual	50	3.9493	0.0790		
Total	79	16.2477			

As expected from the design, sums of squares associated with the mildew fungicide treatment (MildewF) appear in the Block.MainPlot.Subplot stratum, and those involving only the nitrogen rate (NRate) or application method (NForm) appear in the Block.MainPlot stratum. There is no evidence from the variance ratios of any interaction between any of the treatments or of any difference between

population means for the different nitrogen doses or application methods. There is strong evidence of a difference in population yields associated with application of mildew fungicide ($F_{1,50}$ = 102.182, P < 0.001). Examination of the predicted means for this factor (Table S9.6.2) shows that use of fungicide increases yield by 0.635 tonnes/ha with LSD = 0.1262, and this difference summarises the predictive model derived from this trial.

Table S9.6.2 Predicted means (tonnes/ha) for spring barley yield in the absence or presence of Trademorph fungicide. SED = 0.0628, LSD = 0.1262 with α = 0.05, df = 50.

NONE	Trademorph
4.951	5.586

The question asks us to comment on a comparison between solid and liquid forms of fertiliser application. Given that there is no evidence of differences between application methods from testing the NForm main effect, we would not expect a large difference, but we can construct a contrast to test this question directly. The classification of the 5 forms of application as liquid or solid are shown in Table S9.6.3, together with coefficients for a contrast to compare these methods.

Table S9.6.3 Classification of NForm factor levels as liquid or solid and coefficients (not standardized) for LvS contrast to compare liquid vs solid methods.

Factor levels	L	LI	SS	SST	ST
Liquid or solid form?	Liquid	Liquid	Solid	Solid	Solid
LvS contrast coefficients	3/6	3/6	-2/6	-2/6	-2/6

If we add this contrast into the analysis, we can partition sums of squares associated with the NForm factor into a part associated with the factor and a remainder. The ANOVA table with this partitioning applied is shown in Table S9.6.4 – pairs of lines coloured blue each give the partition of the black line immediately above them. There is no evidence in this table of any difference between the liquid and solid forms of fertiliser, and there is no evidence of any difference of behaviour between the liquid and solid forms in combination with any of the other treatments (no interactions with the contrast).

Table S9.6.4 ANOVA table for yield of spring barley in split-plot field trial.

Source of variation	df	Sum of squares	Mean square	Variance ratio	*P*
Block stratum	1	0.5072	0.5072		
Block.MainPlot stratum					
NRate	1	0.2216	0.2216	1.317	0.281
NForm	4	0.8517	0.2129	1.266	0.352
LvS	1	0.0109	0.0109	0.065	0.805
Deviations	3	0.8408	0.2803	1.666	0.243
NRate.NForm	4	0.1985	0.0496	0.295	0.874
NRate.LvS	1	0.0058	0.0058	0.035	0.857
Deviations	3	0.1927	0.0642	0.382	0.769
Residual	9	1.5143	0.1683	2.130	
Block.MainPlot.Subplot stratum					
MildewF	1	8.0709	8.0709	102.182	<0.001
NRate.MildewF	1	0.0865	0.0865	1.096	0.300
NForm.MildewF	4	0.3943	0.0986	1.248	0.303
LvS.MildewF	1	0.2071	0.2071	2.622	0.112
Deviations	3	0.1872	0.0624	0.790	0.505
NRate.NForm.MildewF	4	0.4537	0.1134	1.436	0.236
NRate.LvS.MildewF	1	0.1729	0.1729	2.189	0.145
Deviations	3	0.2808	0.0936	1.185	0.325
Residual	50	3.9493	0.0790		
Total	79	16.2477			