Solution to Exercise 7.1 (Version 1, 7/6/15)

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Exercise 7.1*

A controlled environment experiment to compare the effect of a diet on weight of three aphid species was done using a RCBD with three blocks.

a) What are the null and alternative hypotheses for this experiment?
b) Construct the ANOVA table given that BlkSS = 0.00317, TrtSS = 0.35106 and TotSS = 0.36195.
c) What is the appropriate F-distribution for the treatment variance ratio under the null hypothesis? What is the 5% critical value from this distribution?
d) Would we accept or reject the null hypothesis?

Solution 7.1

a) If we denote the population means for the three species as μ_1, μ_2 and μ_3, then the null hypothesis is

$$H_0: \mu_1 = \mu_2 = \mu_3,$$

i.e. that the population means of the three species are all equal. The alternative hypothesis, H_1, is that the species population means are not all equal.

b) Using the obvious notation, the symbolic form of the model can be written as

Explanatory component:	[1] + Species
Structural component:	Block

There are four lines in the ANOVA table, representing variation due to blocks and species (treatments), background variation and total variation. We have been given the block, treatment and total sums of squares, and can calculate the residual sum of squares as

$$\text{ResSS} = \text{TotSS} - \text{BlkSS} - \text{TrtSS} = 0.36195 - 0.00317 - 0.35106 = 0.00772.$$

To complete the ANOVA table we need to calculate the degrees of freedom for each term. With n = 3 blocks, the block df are computed as BlkDF = $n-1$ = $3-1$ = 2. With t = 3 treatments, we have TrtDF = $t-1$ = $3-1$ = 2. There are $N = t \times n = 3\times3 = 9$ experimental units in total and hence TotDF = $N-1 = 9-1 = 8$. The residual df can be calculated as

$$\text{ResDF} = \text{TotDF} - \text{BlkDf} - \text{TrtDF} = 8 - 2 - 2 = 4.$$

The block, treatment and residual mean squares are then calculated as the block, treatment and residual sum of squares divided by the block, treatment and residual df, respectively. The variance ratio for blocks is computed as the ratio of the block mean square to the residual mean square. Similarly, the variance ratio for treatments is computed as the ratio of the treatment mean square to the residual mean square. The results of all the calculations can be combined to construct the ANOVA table in Table S7.1.1. We can equivalently write this in the form of a multi-stratum ANOVA table as shown in Table S7.1.2 to show better the structure of the design.

Table S7.1.1 ANOVA table for weights of aphids of three species compared using a RCBD with three blocks.

Source of variation	df	Sum of squares	Mean square	Variance ratio
Block	2	0.00317	0.00159	0.821
Species	2	0.35106	0.17553	90.948
Residual	4	0.00772	0.00193	
Total	8	0.36195		

Table S7.1.2 Multi-stratum ANOVA table for weights of aphids of three species compared using a RCBD with three blocks.

Source of variation	df	Sum of squares	Mean square	Variance ratio
Block stratum				
Residual	2	0.00317	0.00159	0.821
Block.Unit stratum				
Species	2	0.35106	0.17553	90.948
Residual	4	0.00772	0.00193	
Total	8	0.36195		

c) Under the null hypothesis (that the species means are equal), the treatment (species) variance ratio has as an F-distribution with 2 numerator and 4 denominator df. The 5% critical value of this distribution is $F_{2,4}^{[0.05]} = 6.944$ (see Appendix B.1).

d) The 5% critical value of the F-distribution with 2 and 4 df is very much smaller than the observed variance ratio, $F_{2,4} = 90.95$. We would therefore reject the null hypothesis. The observed significance level, obtained as the proportion of the F-distribution with 2 and 4 df greater than the observed variance ratio, is $P = 0.00046$. Hence, we conclude that there is very strong evidence ($P < 0.001$) that the null hypothesis is not true, ie. we infer that the species population means are not equal.