

Solution to Exercise 5.1a (Version 1, 26/09/14)

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Exercise 5.1a (Data: courtesy K. Hammond-Kosack, Rothamsted Research)

Obtain the simple and standardized residuals from the ANOVA for the data from Exercise 4.2. Use a scatter plot to compare the simple and standardized residuals. Can you explain the patterns that you see? Are there any potential outliers?

Solution 5.1a

The fitted value for an observation in the j th treatment group is the predicted mean for that group, which is equal to the treatment sample mean. The treatment sample means are (see Solution 4.2):

$$\overline{Grains_{1\cdot}} = 10.83, \overline{Grains_{2\cdot}} = 5.50, \overline{Grains_{3\cdot}} = 5.33, \overline{Grains_{4\cdot}} = 5.50, \overline{Grains_{5\cdot}} = 5.83.$$

The simple residuals are the differences between the observed and fitted values,

$$\hat{e}_{jk} = Grains_{jk} - \overline{Grains_{j\cdot}},$$

and the standardized residuals are the simple residuals divided by their estimated standard error, i.e.

$$r_{jk} = \frac{\hat{e}_{jk}}{SE(\hat{e}_{jk})} = \frac{\hat{e}_{jk}}{s} \sqrt{\frac{n}{n-1}} = \frac{\hat{e}_{jk}}{2.48} \sqrt{\frac{6}{5}} = 0.441 \times \hat{e}_{jk},$$

where $s = \sqrt{s^2} = \sqrt{\text{ResMS}} = 2.48$ and $n = 6$. The simple and standardized residuals for grain numbers obtained from these formulae are shown in Table S5.1.1.

A plot of the simple residuals against the standardized residuals is shown in Figure S5.1.1. Since all the treatments have equal replication, the set of standardized residuals is simply a scaled version of the set of simple residuals and so the points fall on a straight line. The largest standardized residual (for ear 7) is slightly greater than two (2.43), but examination of the distribution of the standardized residuals indicates no outliers.

Table 5.1.1 Simple and standardized residuals from a single factor model for grain numbers.

Ear	Treatment	j	k	$Grains_{jk}$	$\overline{Grains}_{j\bullet}$	\hat{e}_{jk}	r_{jk}
1	A	1	1	11	10.83	0.17	0.07
2	A	1	2	7	10.83	-3.83	-1.69
3	A	1	3	10	10.83	-0.83	-0.37
4	A	1	4	10	10.83	-0.83	-0.37
5	A	1	5	13	10.83	2.17	0.96
6	A	1	6	14	10.83	3.17	1.40
7	B	2	1	11	5.50	5.50	2.43
8	B	2	2	8	5.50	2.50	1.10
9	B	2	3	4	5.50	-1.50	-0.66
10	B	2	4	1	5.50	-4.50	-1.99
11	B	2	5	3	5.50	-2.50	-1.10
12	B	2	6	6	5.50	0.50	0.22
13	C	3	1	9	5.33	3.67	1.62
14	C	3	2	7	5.33	1.67	0.74
15	C	3	3	5	5.33	-0.33	-0.15
16	C	3	4	3	5.33	-2.33	-1.03
17	C	3	5	5	5.33	-0.33	-0.15
18	C	3	6	3	5.33	-2.33	-1.03
19	D	4	1	8	5.50	2.50	1.10
20	D	4	2	5	5.50	-0.50	-0.22
21	D	4	3	4	5.50	-1.50	-0.66
22	D	4	4	5	5.50	-0.50	-0.22
23	D	4	5	6	5.50	0.50	0.22
24	D	4	6	5	5.50	-0.50	-0.22
25	Control	5	1	9	5.83	3.17	1.40
26	Control	5	2	7	5.83	1.17	0.51
27	Control	5	3	5	5.83	-0.83	-0.37
28	Control	5	4	5	5.83	-0.83	-0.37
29	Control	5	5	3	5.83	-2.83	-1.25
30	Control	5	6	6	5.83	0.17	0.07

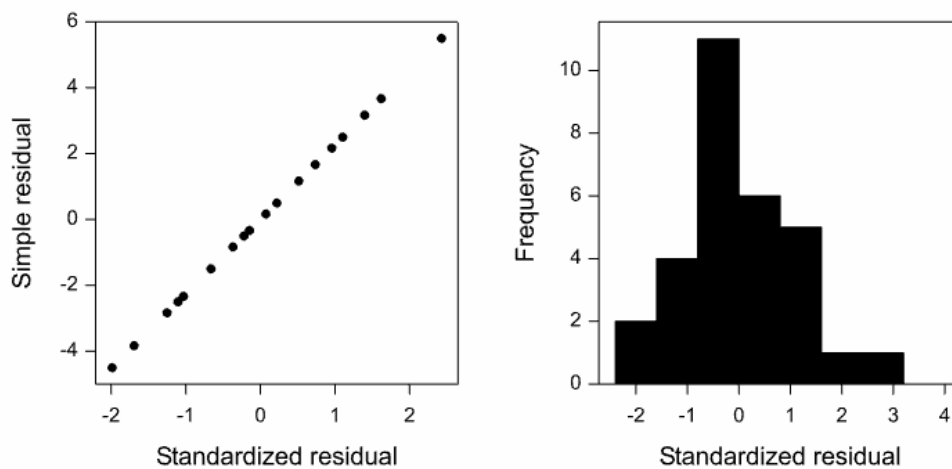


Figure 5.1.1. Scatter plot of simple residuals against standardized residuals with histogram of standardized residuals.